MATHEMATICS.

IMP Revision Problems from last 5 years Board Exam)

2017 - 18

1. Prove that $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$

2. Solve : $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

3. If $x^y + y^x = a^b$ find $\frac{dy}{dx}$

4. If $x = a(\cos t + t\sin t)$ and $y = b(\sin t - \cos t)$, find $\frac{d^2y}{dx^2}$

5. Evaluate : $\int \frac{\sin x + \cos x}{\sqrt{\sin x - \cos x}}\, dx$

6. Evaluate : $\int e^x\left(\frac{x^2+1}{(x+1)^2}\right) dx$

7. Prove that : $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = \pi$

8. Evaluate : $\int_{-1}^{1/2} |x\cos(\pi x)|\, dx$

9. Solve the diff. equ : $y e^{x/y} dx = (x e^{x/y} + y) dy$

10. Solve the diff equ : $(1+y+x^2y)dx + (x+x^3)dy = 0$, where $y=0$, when

11. Solve for x : $2\tan^{-1}(\sin x) = \tan^{-1}(2\sec x)$, $0 \le x < \frac{\pi}{2}$

12. Show that : $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right] = \frac{x+y}{1-xy}$

13. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, Prove that, $\frac{dy}{dx} = \sqrt{\frac{1-y}{1-x}}$

14. If $y = (x+\sqrt{x^2+1})^m$ Show that. $(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2y = 0$

15. Evaluate : $\int x(\log x)^2 dx$

16. Evaluate : $\int \frac{x}{x^3-1}\, dx$

17. Evaluate : $\int_0^{\pi} \frac{x\, dx}{4-\cos^2 x}$

18. Evaluate : $\int \frac{1-x^2}{x(1-2x)}\, dx$

19. Evaluate : $\int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\, dx$

20. Evaluate : $\int \frac{x+2}{\sqrt{(x-2)(x-3)}}\, dx$

21. Evaluate : $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}\, dx$

22. Evaluate : $\int \frac{(x-4)e^x}{(x-2)^3}\, dx$

23. Evaluate : $\int_0^{\pi/2} (2\log\sin x - \log\sin 2$

24. Differentiate

25. Prove that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = (1+a^2+b^2+c^2)$

26. If $A=\begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$ find A^{-1} and hence solve system of equation $2x+y+3z=3$, $4x-y=3$, $-7x+2y+z=5$

27. If $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}$ and $\vec{b}=(6\hat{i}+\lambda\hat{j}+9\hat{k})$ and $\vec{a}\parallel\vec{b}$ find λ.

28. for what values of a and b function f defined as

$$f(x)=\begin{cases} 3ax+b, & \text{if } x<1 \\ 11, & \text{if } x=1 \\ 5ax-2b, & \text{if } x>1 \end{cases}$$ is continuous at $x=1$

29. find the value of λ such that the line $\frac{x-2}{9}=\frac{y-1}{\lambda}=\frac{z+3}{-6}$ is perpendicular to the plane $3x-y-2z=7$

30. Show that the function $f: R \to R$ define by $f(x)=2x^3-7$ for $x \in R$ is bijective.

31. Show that the four points $(0,-1,-1)$, $(4,5,1)$, $(3,9,4)$ & $(-4,$ are coplanar. Also find the equation of the plane containing th

32. A coin is biased so that head is 3 times as likely to occu as tail. If the coin is tossed three times, find the probabi distribution of number of tails.

33. find the equation of the perpendicular drawn from the point $P(2,4,-1)$ to the line $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{4}$

34. find the equation of the plane passing through the point $(1,1,1)$ and containing the line $\vec{r}=(-3\hat{i}+\hat{j}+5\hat{k})+\lambda(3\hat{i}-\hat{j}+5\hat{k})$

35. Show that the maximum volume which can be inscribed in a cone of height h and semivertical angle α is $\frac{4}{27}\pi h^3 t$

36. Using elementary row operation find inverse of $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

37. Find the points on the curve $y=x^3$ at which the slope of the tangent is equal to the y-coordinate of the point

38. Find general solution of differential equation

39. From a lot of 10 bulbs, which includes 3 defective, a sample of 2 bulbs is drawn at random. Find the probabil distribution of the number of defective bulbs.

40. Using integration, find the area of the region bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

41 Show that the right circular cylinder open at the top, an of given surface area and maximum volume is such tha its height is equal to the radius of the base.

42- Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 18$.

43 One kind of cake requires 300g of flour and 15g of fat, another kind of cake requires 150g of flour and 30g of fat. ~~[illegible]~~ Find the maximum number of cakes which can be made from 7.5kg of flour and 600 gm of fat, assuming that there is no shortage of ingredients used in making the cakes. Make it as an L.P.P and solve it graphic

44. Show that the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar. Also find the equation of the plane.

45 If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, show that $A^2 - 5A - 14I = 0$. Hence find A^{-1}

46 Verify Rolle's Theorem for the function f given by
$f(x) = e^x(\sin x + \cos x)$ on $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$.

47. Prove that
$$\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$$

48. If $y = e^{a\sin^{-1}x}$, $-1 \leq x \leq 1$, then show that
$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$

PN. -3)

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49 prove that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = (1+a^2+b^2+c^2)$

50. from a pack of 52 cards, a card is lost. from the remaining 51 cards, two cards are drawn at random (without re...) are found to be both diamonds. what is the probability that the lost card was a card of heart.

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